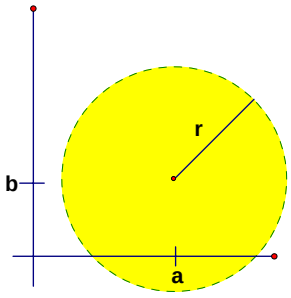
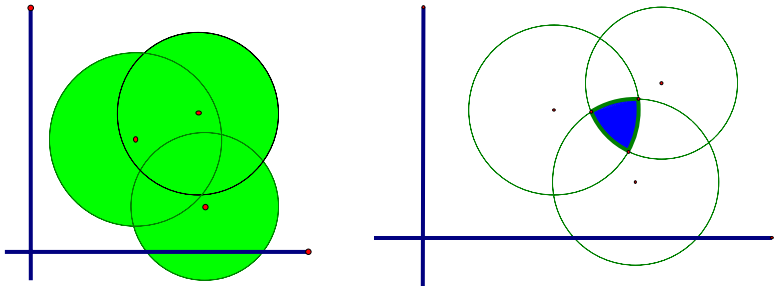
Math History for Precalculus

Felix Hausdorff

1. When did Felix Hausdorff (HOWS dorf) live? *1868–1942*
2. Where was he from? *Germany*
3. What branch of mathematics did he research? *(point set) topology*
4. Which of his books established the subject as a separate discipline? *Grundzüge der Mengenlehre (Basic Features of Set Theory)*
5. When was this book published? *1914*
6. Draw a picture of this set: $A = \left\{ (x, y) \middle| (x-a)^2 + (y-b)^2 < r^2 \right\}$. This is called an open disk.



7. Imagine the union of all open disks. What would you get? *the plane*
8. Draw a set that is not an open disk, but is a union of three open disks. Draw another set that is an intersection of 3 open disks.



9. Hausdorff defined the important concept of a topological space. A topological space is a set with a family of subsets that satisfy three properties. Name them (questions 6–8 provide an example). *1) The family of subsets must contain the whole set (the plane in the example) and the empty set. 2) The family of subsets must contain*

all possible unions of these sets. 3) The family of subsets must contain all intersections of a finite number of these sets.

10. A topological space is called a Hausdorff space if, given any pair of points, you can draw disjoint open sets around them. As an example, give equations of open disks around points (5,2) and (4,3) that do not overlap. What is the intersection of the disks?

(Any values of r will work as long as their sum does not exceed $\sqrt{2} \approx 1.4$*, the distance between the two points.)*

$A = \left\{ (x, y) \middle| (x-5)^2 + (y-2)^2 < 1 \right\}$ *(note: r = 1);*

$A = \left\{ (x, y) \middle| (x-4)^2 + (y-3)^2 < \frac{1}{9} \right\}$ *(note: r =* $\frac{1}{3}$*)*